R2-35 Uncertainties in Distribution Temperature Determination

Report to CIE Division 2, 17 May 2005

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Terms of Reference

To investigate the potential effect of a change to the definition of distribution temperature to include a statement regarding weighting the spectral distribution values by the uncertainty of the measurements at each wavelength.

Background

Radiation from an incandescent tungsten filament lamp typically has a relative spectral distribution very similar to that of a blackbody radiator. Thus, it is convenient and useful in practice to describe the distribution by a single number, the temperature of the closest blackbody distribution. The most common implementation of this concept is correlated colour temperature, which is defined as the temperature of the blackbody whose colour is closest to that of the test radiation. However, in some applications, the quantity distribution temperature is more useful. Distribution temperature is the temperature of the blackbody radiator whose relative spectral distribution is closest to that of the test radiation. This is a physical definition that is useful in applications that do not involve vision and colour.

The precise definition recommended by the CIE in Publication 114/4-1994 is:

> The distribution temperature of a source in a given wavelength range, λ_1 to λ_2, is the temperature, T_D, of the Planckian radiator for which the following integral is minimized by adjustment of a and T:
>
> $$\int_{\lambda_1}^{\lambda_2} [1 - S_t(\lambda) / aS_b(\lambda, T)]^2 \mathrm{d}\lambda \qquad (1)$$
>
> where λ is the wavelength, $S_t(\lambda)$ is the relative spectral distribution of the radiation being considered, $S_b(\lambda,T)$ is the relative spectral distribution of the Planckian radiator at temperature T, and a is a scaling factor.

Issue

During the meeting of TC 2-43 (Determination of measurement uncertainties in photometry) held on 9 June 2004, the TCC (Sauter, DE) proposed that uncertainties of the spectral values should be considered in the calculation of distribution temperature; i.e., the differences between two curves should be weighted by the inverse square of the uncertainty value of each point in order to obtain the best estimate based on statistical theory. Ohno (US) raised a concern that this would be a change of the definition given in CIE 114/4, and that, if this change was recommended, it would be possible to obtain two different answers from the same spectral data, depending on whether weight is applied or not. This would cause serious confusion. Robertson (CA) commented that the definition should not change because, in a definition, each quantity should be assumed to have zero uncertainty. However, he added that it might be logical to take account of the uncertainties when calculating the best estimate of distribution temperature based on real measured data. This would mean that the definition would remain unchanged but that the recommended calculation method would be to minimize a modified integral:

$$\int_{\lambda_1}^{\lambda_2} \frac{1}{u_{\mathrm{st,rel}}^2(\lambda)} \left[1 - \frac{S_{\mathrm{t}}(\lambda)}{aS_{\mathrm{b}}(\lambda,T)}\right]^2 \mathrm{d}\lambda \tag{2}$$

where $u_{\mathrm{st,rel}}(\lambda)$ is the relative standard uncertainty of the measurement of $S_{\mathrm{t}}(\lambda)$.

As in most photometric calculations, the integral would normally be approximated by summation at equal wavelength intervals:

$$\sum_{i=1,n} [1 - S_{\mathrm{t},i} / aS_{\mathrm{b},i}(T)]^2 \tag{3}$$

for the unweighted expression (1), and

$$\sum_{i=1}^{n} \frac{1}{u_{\mathrm{st,rel},i}^2} \left[1 - \frac{S_{\mathrm{t},i}}{aS_{\mathrm{b},i}(T)}\right]^2 \tag{4}$$

for the weighted expression (2).

However, in the D2 meeting on 11 June 2004, Ohno pointed out that this method would yield errors in certain common situations. He showed a simulation involving a common situation in which $S_t(\lambda)$ followed a blackbody curve for wavelengths greater than 450 nm but fell below the blackbody for wavelengths less than 450 nm. Using the unweighted expression (3), the distribution temperature of this source is slightly less than that of the blackbody that it follows above 450 nm. However, because, in most spectroradiometric systems, the uncertainty of measurement is greater at low wavelengths (400 to 450 nm) than it is in most of the visible spectrum, the deviations from a blackbody will be given less weight by expression (4), resulting in a distribution temperature that is closer to that of the underlying blackbody. Thus, use of expression (4) leads to an error. Such errors will occur whenever the measurement uncertainty is greater in spectral regions where a source deviates from a blackbody distribution than it is in other regions.

Discussion

It has been suggested that the “GUM” (the 1993 ISO Guide to the Expression of Uncertainty in Measurement) requires that weights be used. However, this Reporter can find no such requirement in the GUM and notes, as an analogy, that in the analysis of key comparisons conducted under the CIPM MRA, both weighted and unweighted means have been used depending on specific attributes of the comparison.

Two points support the contention that weights should not be used in the calculation of distribution temperature:

1. The idea behind the concept of distribution temperature is that it provides a single number that can be used to specify a spectral power distribution. This implies that all

parts of the spectrum should be treated equally and that no parts should be “penalized” because of characteristics of particular measurement techniques.

2. A fundamental principle of metrology is that a measurand should be defined in such a way that its value does not depend on the method of measurement. The use of weights in the definition or the recommended calculation method would contradict this principle.

Recommendation

The recommendation of this Reporter is that, even though the use of weights, as in expression (4), may be justified from a theoretical statistical point of view, weights should not be used in practice because they can introduce small errors in certain common situations.